Faculty of Science and Technology
Tokushima University

Mathematical Analysis of Nonlinear Phenomena

Professor Kosuke Ono

1. Linear Dissipative Wave Equation:

$$\begin{cases} (\Box + \partial_t)u = 0 \quad \text{in} \quad \mathbb{R}^N \times (0,\infty) \\ (u, \partial_t u)|_{t=0} = (u_0, u_1) \quad \text{in} \quad \mathbb{R}^N \end{cases}$$

[Energy Decay in Energy Spaces]

$(u_0, u_1) \in H^1(\mathbb{R}^N) \times L^2(\mathbb{R}^N)$

$\Longrightarrow$

$E(u(t), \partial_t u(t)) \le C(1+t)^{-1}$

[Sharp Decay] $m \ge 0$, $N = 2n$ or $2n+1$

$(u_0, u_1) \in (H^{m+1}(\mathbb{R}^N) \cap W^{n,1}(\mathbb{R}^N)) \times (H^m(\mathbb{R}^N) \cap W^{n-1,1}(\mathbb{R}^N))$

$\Longrightarrow$

$\|\partial_t^k \nabla_x u(t)\|_{L^q(\mathbb{R}^N)} \le C(1+t)^{-k-\frac{|\beta|}{2}-\frac{N}{2}(1-\frac{1}{q})}$

$(1 \le q \le 2,\ 0 \le k + |\beta| \le m,\ k \ne m)$

2. Nonlinear Degenerate Dissipative Kirchhoff Equation:

$$\begin{cases} \rho \partial_t^2 u - \left(\int_\Omega |\nabla_x u(x,t)|^2 dx\right)^\gamma \Delta_x u + \partial_t u = 0 \quad \text{in} \quad \Omega \times (0,\infty) \\ (u, \partial_t u)|_{t=0} = (u_0, u_1) \quad \text{in} \quad \Omega, \quad \Omega : \text{bounded in } \mathbb{R}^N \\ u(x,t) = 0 \quad \text{on} \quad \partial\Omega \times (0,\infty) \end{cases}$$

[Optimal Decay] $\rho > 0$, $\gamma > 0$

$(u_0, u_1) \in (H^2(\Omega) \cap H_0^1(\Omega)) \times H_0^1(\Omega)$, $u_0 \ne 0$, $\rho \ll 1$

$\Longrightarrow$

$C^{-1}(1+t)^{-\frac{1}{\gamma}} \le \|\nabla_x^k u(t)\|^2_{L^2(\Omega)} \le C(1+t)^{-\frac{1}{\gamma}} \quad (k = 0, 1, 2)$

3. Vlasov-Poisson-Fokker-Plank System:

$$\begin{cases} \partial_t f + v \cdot \nabla_x f + E \cdot \nabla_v f - \Delta_v f = 0 \quad \text{in} \quad \mathbb{R}^N \times \mathbb{R}^N \times (0,\infty) \\ -\Delta_x U = \gamma \int_{\mathbb{R}^N} f(x,v,t)\, dv, \quad E = -\nabla_x U \\ f(x,v,0) = f_0(x,v) \quad \text{in} \quad \mathbb{R}^N \times \mathbb{R}^N, \quad \gamma = \pm 1 \end{cases}$$

[Asymptotic Behavior] $1 \le p \le \infty$

$f_0 \in L^p(\mathbb{R}^N \times \mathbb{R}^N)$, $\|f_0\| \ll 1$

$\Longrightarrow$

$$\|\nabla_x^\alpha \nabla_v^\beta f(t) - \nabla_x^\alpha \nabla_v^\beta h(t)\|_{L^q(\mathbb{R}^N \times \mathbb{R}^N)} \le Ct^{-\frac{1}{2}(3|\alpha|+|\beta|)}(1+t)^{-\frac{1}{2}-2N(1-\frac{1}{q})} \quad (1 \le q \le p)$$

where h is the solution of the linear Fokker-Plank system

Content:

By using the functional analysis approach, I study on the structure of solutions and the decay estimate of energy for nonlinear PDEs that describe the nonlinear phenomena. In a study of nonlinear equations, detailed analysis of the corresponding linear equation becomes essential. I examine the relationship of the nonlinearity of the equations and the functional spaces to which the solutions belong, and I investigate on global solvability in time of solutions or blow-up problems for PDEs. In addition, I research on decay estimates the energy function and the derivatives.

Nonlinear degenerate dissipative Kirchhoff equations are nonlinear PDEs that describe the nonlinear wave phenomena. Solutions of these equations have decay estimates of the same polynomial order from above and blow.

Vlasov-Poisson systems are basic equations that describe plasma phenomena, in particular, the solution of the Vlasov-Poisson-Fokker-Plank system is asymptotic to the solution of the corresponding linear Fokker-Plank system.

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E-mail : k.ono@tokushima-u.ac.jp
Tel. +81-88-656-7218
Fax : +81-88-656-7218
HP : http://www-math.ias.tokushima-u.ac.jp/