Faculty of Science and Technology
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Global Asymptotics of the Painlevé equations

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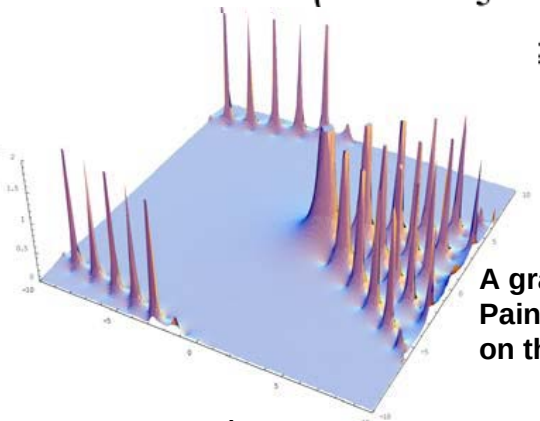
Elliptic Asymptotics of the first Painlevé equations

$$y'' = 6y^2 + x$$

$$y(x) \sim |x|^{1/2}\wp\left(\frac{4}{5}e^{i\varphi}|x|^{5/4} - t(\varphi, s); g_2(\varphi), g_3(\varphi)\right) + O(|x|^{3/4}),$$

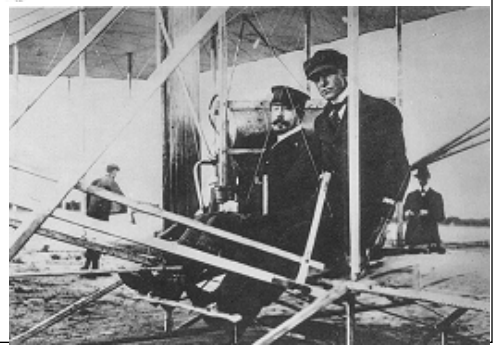
$$t(\varphi, s) = \frac{1}{2\pi i}\left(\omega_a(\varphi)\log(is_{2-2k}) + \omega_b(\varphi)\log\frac{s_{5-2k}}{s_{2-2k}}\right)$$

$$x \in D_k(\varphi, \varepsilon, s) = \left\{x \in \mathbb{C}; \frac{(3+2k)\pi}{5} + \varepsilon \leqq \varphi \leqq \frac{(5+2k)\pi}{5} - \varepsilon\right\}$$



A graph of the first Painlevé transcendents on the complex domain

Paul Painlevé (left) was a former prime minister of France. He is the first mathematician who flew on the airplane. He was a passenger of Wilber Wright (right) on 1908.



Content:

It is important to study connection problems of solutions of differential equations between two points in many fields of mathematical sciences. The global study on ordinary linear differential equations is still developing.

Paul Painlevé studied nonlinear differential equations with second order, which have no movable branch points (so called **the Painlevé property**). He and his pupil, Gambier, classified all of such equations in six types around 1900. Many nonlinear equations appeared in physics has the Painlevé property, and we can solve connection problems on such equations. We expect that the **Painlevé equations** play the same important role in nonlinear analysis as the Bessel functions or hypergeometric functions play in linear equations. The Painlevé equations are also obtained by monodromy preserving deformations. We can show the correspondence between global data of linear equations and local data of the Painlevé functions and we can study nonlinear connection problems or the nonlinear Stokes phenomenon on the Painlevé differential or difference equations.

Keywords : Classical Analysis, the Painlevé equations, monodromy problems

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