A comparison principle and a strong comparison principle of nonlinear partial differential equations

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Let $\Omega \subset \mathbf{R}^N$ (domain). We consider the following PDE.

(1.1) $$F(x, Du(x), D^2u(x)) = 0 \quad \text{in} \quad \Omega,$$

$$Du = \left(\frac{\partial u}{\partial x_1}, \cdots, \frac{\partial u}{\partial x_N}\right), \quad D^2u = \left(\frac{\partial^2 u}{\partial x_i \partial x_j}\right) \ \text{(Hessian of } u\text{)}.$$

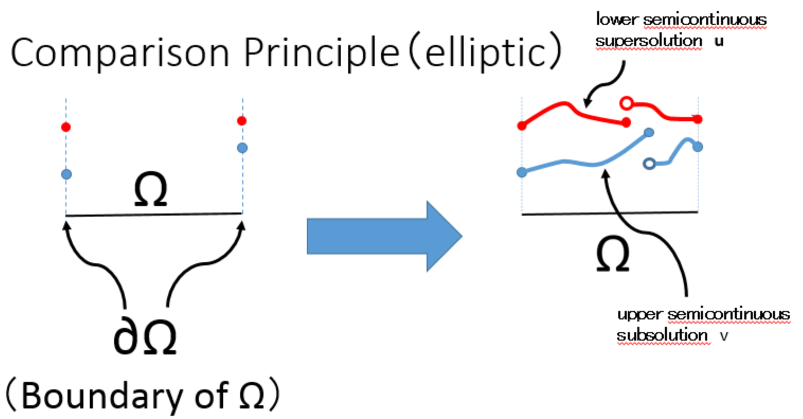
$D^2u \in \mathbf{S}^N$ ($N \times N$ real symmetric matrices)

Example of (1.1) (**the minimal surface equation for graph.**)

(1.2) $$-\sqrt{1+|Du|^2}\,\mathrm{div}\left(\frac{Du}{\sqrt{1+|Du|^2}}\right) = 0 \quad \text{in} \quad \Omega.$$

Example of (1.1)(**the prescribed mean curvature equation.**)
For a given function $H \in C^1(\Omega)$,

(1.3) $$\mathrm{div}\left(\frac{Du}{\sqrt{1+|Du|^2}}\right) = NH \quad \text{in} \quad \Omega.$$



Content:

I am interested in the study of a comparison principle and a strong comparison principle for semicontinuous solutions of nonlinear partial differential equations.

As partial differential equations I considered the minimal surface equation, the prescribed mean curvature equation, the level set equation of the mean curvature flow equation, the level set equation of an anisotropic curvature equation and p-Laplace diffusion equation. As well known the above equations are degenerate and singular. Usually for such equation, we cannot expect existence of classical solutions. So I will consider such equations with viscosity solutions.

For elliptic equations:

Comparison principle: Let u be a lower semicontinuous supersolution, and let v be a upper semicontinuous subsolution. On the boundary of the domain we considered if u is greater than or equal to v, then it holds in the whole domain.

Strong comparison principle: Assume in the whole domain u is greater than or equal to v. If u touches v in a interior point of the domain, then u is equivalent to v in the whole domain.

Keywords: partial differential equation, viscosity solution

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