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Limit Cycles for 3D Competitive Lotka-Volterra systems

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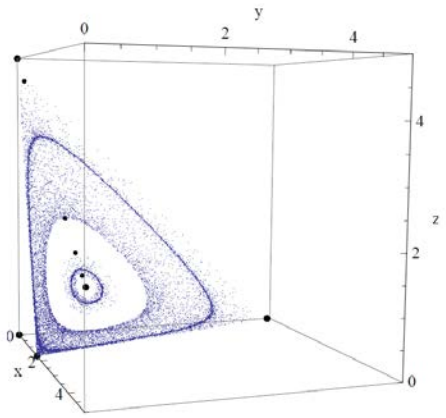

Fig.1 Phase Portrait of Zeeman's class 27

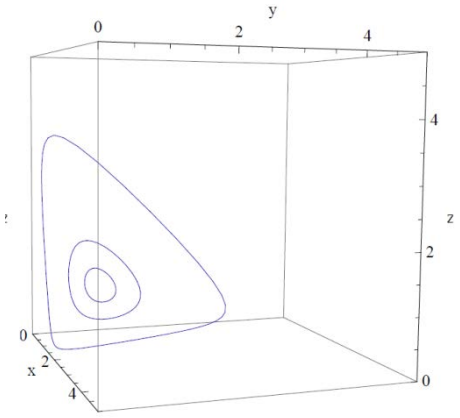

Fig.2 Limit cycles of Zeeman's class 27

Content:

2D Lotka-Volterra equations cannot have limit cycles. That is, except for conservative systems, the limit set consists of equilibria only. On the other hand, 3D Lotka-Volterra systems allow various types of complicated dynamics.

As long as we restrict 3D competitive Lotka-Volterra systems, the possibility of the dynamics is limited. Hirsch showed that competitive systems have the order preserving property, and there is an invariant manifold (called the carrying simplex) which attracts all orbits except for the origin. Thus, in 3D competitive systems, the Poincare-Bendixson theorem holds, and therefore the limit set consists of equilibria, limit cycles and heteroclinic orbit only. Zeeman has divided all possible phase portraits of 3D competitive Lotka-Volterra systems into 33 classes and showed that six classes can have the Hopf bifurcation. Hofbauer and So constructed an example with two limit cycles in Zeeman's class 27.

In this study, we present a concrete example with multiple limit cycles for 3D competitive Lotka-Volterra systems. For instance, we obtain an example with three limit cycles in Zeeman's class 27 as shown in figures.

Keywords: differential equations, Hopf bifurcation, limit cycles

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