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Nonlinear partial differential equations of elliptic type: Qualitative theory

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Example

$$x = (x_1, x_2, \ldots, x_N) \in \Omega \subset \mathbb{R}^N$$

$$u = u(x) = u(x_1, x_2, \ldots, x_N)$$

- Variational problem

$$I(u) = \int_\Omega \{\Phi(|\nabla u|) - \lambda F(u)\}\, dx$$

- Nonlinear eigenvalue problem

$$-\operatorname{div}(\phi(|\nabla u|)\nabla u) = \lambda f(u) \text{ in } \Omega$$
$$u = 0 \text{ on } \partial\Omega$$

Content:

Boundary value problems of partial differential equations arise in a variety of contexts in mathematical sciences, for example, geometry, physics, mechanics, life sciences, economics and so on. In a long history of mathematical analysis, linear differential equations have been very fundamental and important in this area. Moreover, the theory of nonlinear equations is also interesting and in progress. Recently, topological and variational methods are systematically studied by many researchers and developed to a powerful tool in the theory of nonlinear partial differential equations. Our interest here is the qualitative theory of quasilinear elliptic differential equations.

- Boundary value problem
- Calculus of variations
- Existence of solutions
- Uniqueness and multiplicity of the solutions
- Dependence on the parameter
- Asymptotic properties
- A priori estimates and regularity estimates

In particular variational method is useful.

Keywords: mathematical analysis, nonlinear differential equations, boundary value problems, qualitative theory

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