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Analysis of the stationary Navier-Stokes equations

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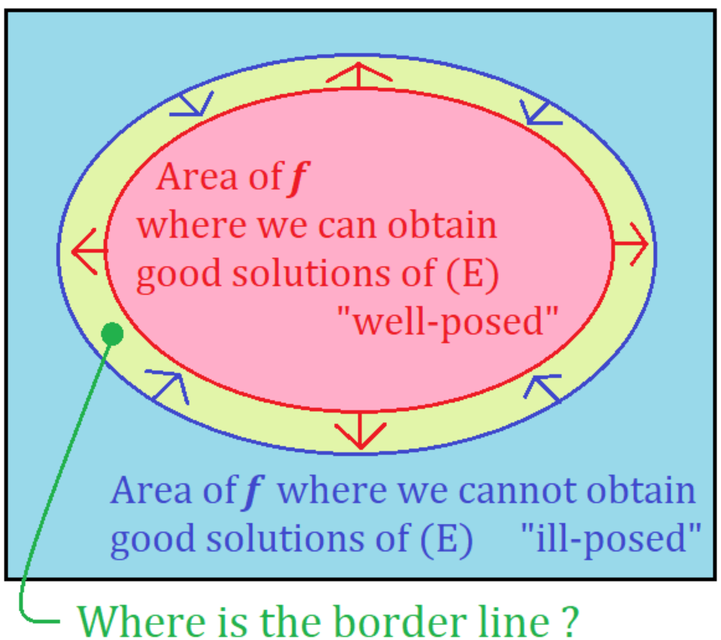
Stationary Navier-Stokes equations:

$$\begin{cases} -\Delta \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \nabla p = \boldsymbol{f} \\ \operatorname{div} \boldsymbol{u} = 0 \end{cases} \quad \text{(E)}$$

$\boldsymbol{u}$: flow velocity, p: pressure ← unknown

$\boldsymbol{f}$: external force ← given

Image: Function spaces of $\boldsymbol{f}$



Content:

I am interested in the stationary Navier-Stokes equations, which describes the behavior of a fluid with no time variation of flow velocity. The purpose is to find the borderline between the well-posedness (existence, uniqueness, and continuous dependence of solutions for given external forces) and ill-posedness in terms of function spaces for solutions and external forces.

In the case of the two-dimensional whole space, the analysis of this equation is extremely difficult (due to a phenomenon known as Stokes' paradox). However, there is a few previous studies on the well-posedness around special solutions (e.g., uniform, symmetric, and rotational flows) and the ill-posedness around a trivial solution (zero). Based on these studies, I aim to generalize the conditions for both well-posedness and ill-posedness, and to construct a systematic analysis method for the two-dimensional case.

Keywords : fluid dynamics, partial differential equations, functional analysis

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