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# Derived Categories of Commutative Rings

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<図表>

Tensor Triangular Geometry : Balmer (2005)

$$X \cong \mathrm{Spec}_{\otimes}(\mathrm{D}^{\mathrm{pf}}(X))$$

Forgetting the tensor structure

Triangular Geometry : Matsui (2021)

$$X \subseteq \mathrm{Spec}_{\triangle}(\mathrm{D}^{\mathrm{pf}}(X))$$

The left-hand sides contain geometric information on original Noetherian scheme X and the right-hand sides contain (tensor) triangulated category structure. Hence the above results connect them.

Content:

One of the most classical and important problems in commutative algebra is to classify all modules up to isomorphisms. This problem is quite difficult and it is known to be hopeless in general.

On the other hand, classifying subcategories (certain classes of objects) of a given category is a more manageable approach than classifying objects. So far, there are many results in this approach.

My research aims to connect geometric information on a given commutative ring R or more generally a scheme X and triangulated category structure of their derived categories using classifications of subcategories. In 2005, Balmer developed ``tensor triangular geometry" and proved that the tensor triangulated category structure of the perfect derived category $\mathrm{D}^{\mathrm{pf}}(X)$ determines the original Noetherian scheme X. However, tensor structures are too strong in general and there are various important examples of triangulated categories without natural tensor structures.

Recently, I have generalized his theory without using tensor structure. As Balmer's theory is, this theory is expected to apply in many areas in mathematics such as algebraic geometry, representation theory of groups, stable homotopy theory, and so on.

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