Faculty of Engineering
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Applications of Number Theory and Algebraic Systems

Professor Hiroki Takahashi

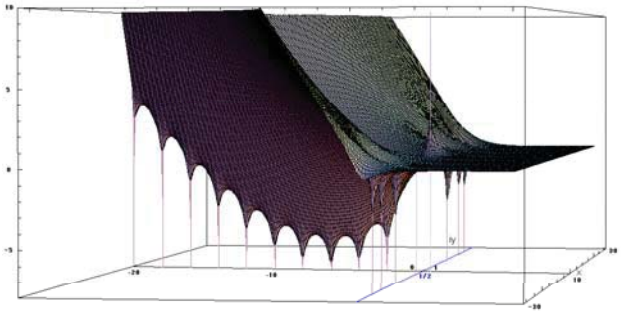

Fig.1 log|ζ(s)| (ζ(s): Riemann zeta function)

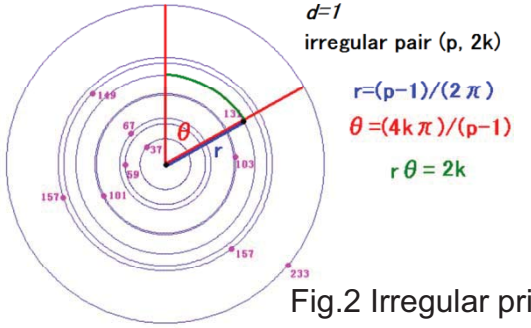

Fig.2 Irregular primes and indices

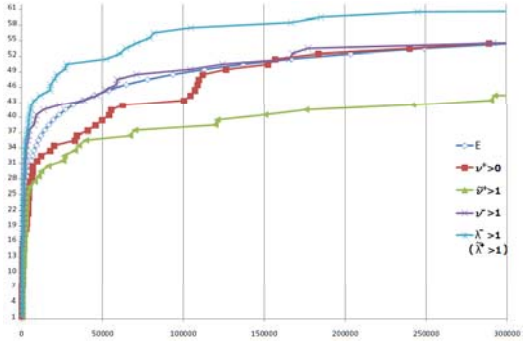

Fig.3 The number of exceptional primes

Content:

The main subject of our research is the ideal class groups of algebraic number fields. We have particularly investigated Greenberg's conjecture and Vandiver's conjecture on the class numbers of real cyclotomic fields by using computers. Furthermore, we are also interested in new applications of algebraic systems such as algebraic number fields and elliptic curves, which have strong connections with cryptography.

A lot of mathematicians have been interested in Riemann zeta function (cf. Fig.1). Its special values have deep relations with the ideal class groups of cyclotomic fields (cf. Fig.2). These relations are expressed as correspondences of the class numbers of real cyclotomic fields and the indices of their circular units in full ones.

Greenberg's conjecture states that their p-parts are bounded in the Zp-extension. Moreover, Vandiver's conjecture states that they are trivial for p-cyclotomic fields. We have been studied these conjectures by using arithmetic special elements such as cyclotomic units, Gauss sums, p-adic L-functions and auxiliary prime numbers. As results, we could find a lot of examples for which Greenberg's conjecture holds, and a lot of exceptional prime numbers for the Iwasawa invariants (cf. Fig.3).

Keywords : algebraic number field,
class number, elliptic curve, cryptography

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Numerical Computation for Population Pharmacokinetics

Professor Toshiki Takeuchi

$$S = 2\omega_c^2 \sum_{j=1}^{m} \log c(t_j, \mathbf{x}) + \sum_{j=1}^{m} \frac{(c_j - c(t_j, \mathbf{x}))^2}{\{c(t_j, \mathbf{x})\}^2} + \omega_c^2 \sum_{i=1}^{n} \frac{(x_i - \mu_i)^2}{\omega_i^2 \mu_i^2}$$

(a) Objective function in nonlinear optimization problem

$C(t) = c(t, V_d, V_{max}, K_m)$: Concentration

$$\begin{cases} \dfrac{dX_a(t)}{dt} = -k_a X_a(t) \\ \dfrac{dC(t)}{dt} = \dfrac{F\, k_a\, X_a}{V_d} - \dfrac{V_{max}\, C}{V_d(K_m + C)} \end{cases} \qquad t_i \leqq t < t_{i+1} \qquad (i = 1, 2, \cdots)$$

$$X_a(t_i) = \begin{cases} D_1 \\ D_i + \lim\limits_{t \to t_i - 0} X_a(t) \end{cases} \qquad C(t_i) = \begin{cases} 0 & i = 1 \\ \lim\limits_{t \to t_i - 0} C(t) & i \geqq 2 \end{cases}$$

(b) Differential equations(phenytoin)

Fig. 1 Bayesian estimation for pharmacokinetics

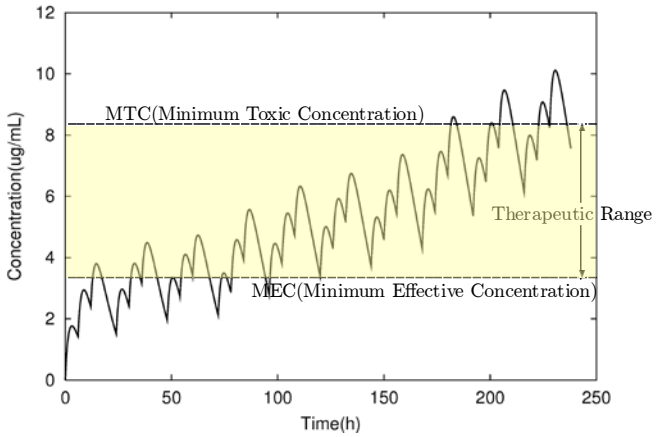

Fig. 2 Profile of the concentration and the therapeutic range

Content:

Pharmacokinetics plays an important role in efficacy and safety pharmacotherapy. The estimation of individual pharmacokinetic parameters from a few concentration data is desirable in quick therapy. Bayesian estimation using the population pharmacokinetic parameters is useful for the estimation of individual pharmacokinetic parameters. Here, population pharmacokinetic parameters mean statistic including average, variance and correlation coefficient. The numerical calculation of nonlinear optimization is essential to Bayesian estimation or computation for population pharmacokinetic parameters. In addition, the theoretical value of concentration data may be given with a nonlinear differential equation. The stable computation in nonlinear optimization for pharmacokinetics is difficult because of the strong nonlinearity. I am developing a stable and high-precision numerical method for nonlinear optimization problem in population pharmacokinetics and Bayesian estimation.

Keywords: Bayesian estimation, Nonlinear optimization

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NMR study on Ion Dynamics in Electrode Materials for Ion secondary battery

Professor Koichi Nakamura

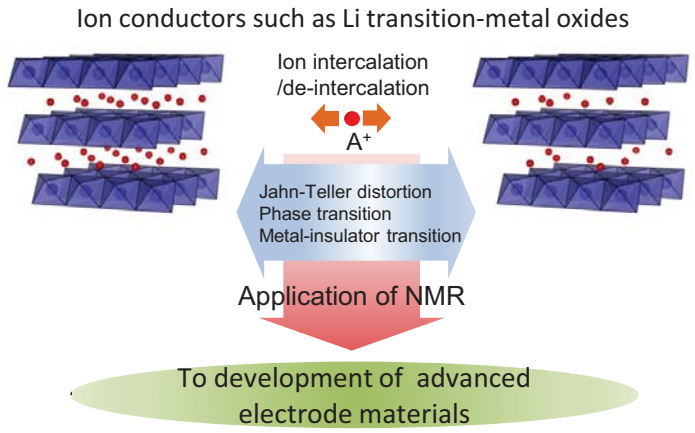
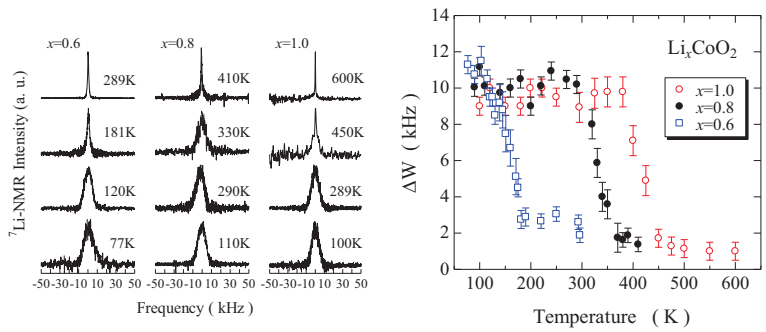


Fig.1 An approach to understanding of ion dynamics with microscopic technique.



(a)^{7}Li-NMR spectra (b) Line widths against temperature

Fig.2 Temperature dependence of ^{7}Li-NMR spectra in LixCoO2. [1]

[1] K.Nakamura *et al.*, Solid State Ionics 177(2006)821.

Content:

Energy device materials such as ion secondary battery and fuel cell are key materials for sustainable energy resource. Conventional electrical measurement technique is microscopically inadequate to study electrochemical reaction. In special, microscopic insights for ion intercalation/de-intercalation are required to develop electrode materials for advanced battery with highly controlled structure.

NMR is a powerful tool to study local ion dynamics and electronic state in electrode materials because of probing nucleus directly. ^{7}Li-NMR spectra are dependent on Li content and temperature as shown in Fig. 2. These changes in spectra are strongly connected with Li^{+} ion motion in Li_xCoO_2 and are microscopically able to answer electrochemical behaviors of electrode materials.

We aim to understand diffusion mechanism and ionic conducting behavior in various ion conductors with NMR technique.

Keywords : Ion conductivity, NMR, Ion secondary battery

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Microscopic Study of Magnetism in Transition Metal Oxides

Associate Professor Yu Kawasaki

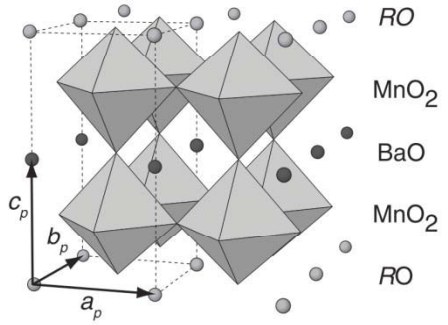

Fig.1 Crystal structure of A-site ordered $RBaMn_2O_6$

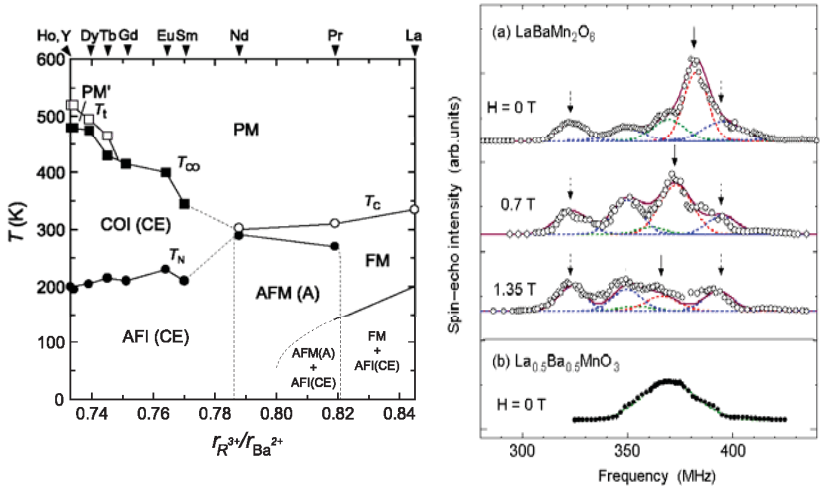

Fig.2 (left) Phase diagram of $RBaMn_2O_6$
(right) Mn-NMR spectra of $LaBaMn_2O_6$

Content:

Transition metal oxides with perovskite structure and their derivatives have been intensively studied in terms of technological application as well as fundamental physics, because of their rich variety of electromagnetic properties, such as high-T_c superconductivity in copper oxides and colossal magnetoresistance in manganese oxides. However, the mechanisms of these physical phenomena are not yet well understood.

To clarify these issues, we study magnetic properties of transition metal oxides by NMR and μSR from a microscopic point of view. For example, we investigate the A-site randomness effect in Ba-based manganites. In this work, we investigate the magnetically ordered states of the A-site ordered $RBaMn_2O_6$ (R: rare earth atoms), which are free from A-site randomness due to the layer-type ordering of R and Ba atoms at the A-site of the structure (Figs.1 and 2).

Keywords: Strongly correlated electron systems,
Magnetism, Superconductivity,
Magnetic Resonance

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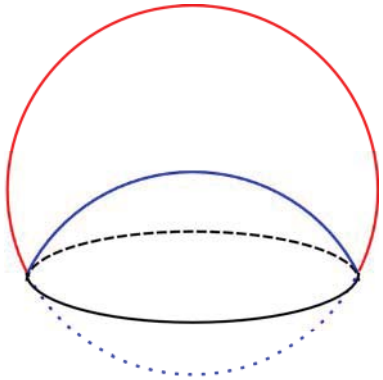
Non-linear Partial Differential Equation

Associate Professor Atsuhito Kohda



Content:

One main theme is elliptic partial differential equation, in particular, equation of constant mean curvature. It is a very interesting subject. It is known that there are two kind of solutions for it.
One is a small solution, which is stable one, and another is a large solution which is unstable.
It is an important problem if there is third solution or not under suitable conditions.
Another topic is hyperbolic systems of conservation laws, for example, Burgers' equation. It is well-known that there shall be discontinuous solution. But non-linearity prevents one from applying modern mathematical techniques such as distribution solution. So one needs to study discontinuity with a direct method, like measure theoretic notion.
At present, it has a solution under very restrictive condition so it will be interesting to consider more weak condition which assure an existence of solution.

Keywords：Palais-Smale condition, Burgers' equation
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Explicit study of modular forms

Associate Professor Yoshinori Mizuno

$\{A(T)\}$: Sequence

↓↑

$\mathrm{F}(\mathrm{Z}) = \sum_T A(T) e^{2\pi i\, tr(TZ)}$

: Fourier series (modular form)

↓↑

$\mathrm{D}(\mathrm{s}) = \sum_T \frac{A(T)}{|T|^s}$

: Dirichlet series (analytic continuation and functional equation)

Find new unexpected interplay and their applications (complementary each other)

Content:

Objects

To study modular forms and its Dirichlet series. Namely, I am interested in their interplay and applications complementary each other.

Motivation

Arithmetic nature of the Fourier and Dirichlet coefficients.

Accordingly, I am interested in several aspects of modular forms.

Sales points

(1) explicit computations as far as possible

(2) use of analysis (integral transforms, special functions and harmonic analysis) to get arithmetic consequence

(3) including higher degree case

(4) familiarity with Eisenstein series

Keywords : modular forms, Dirichlet series

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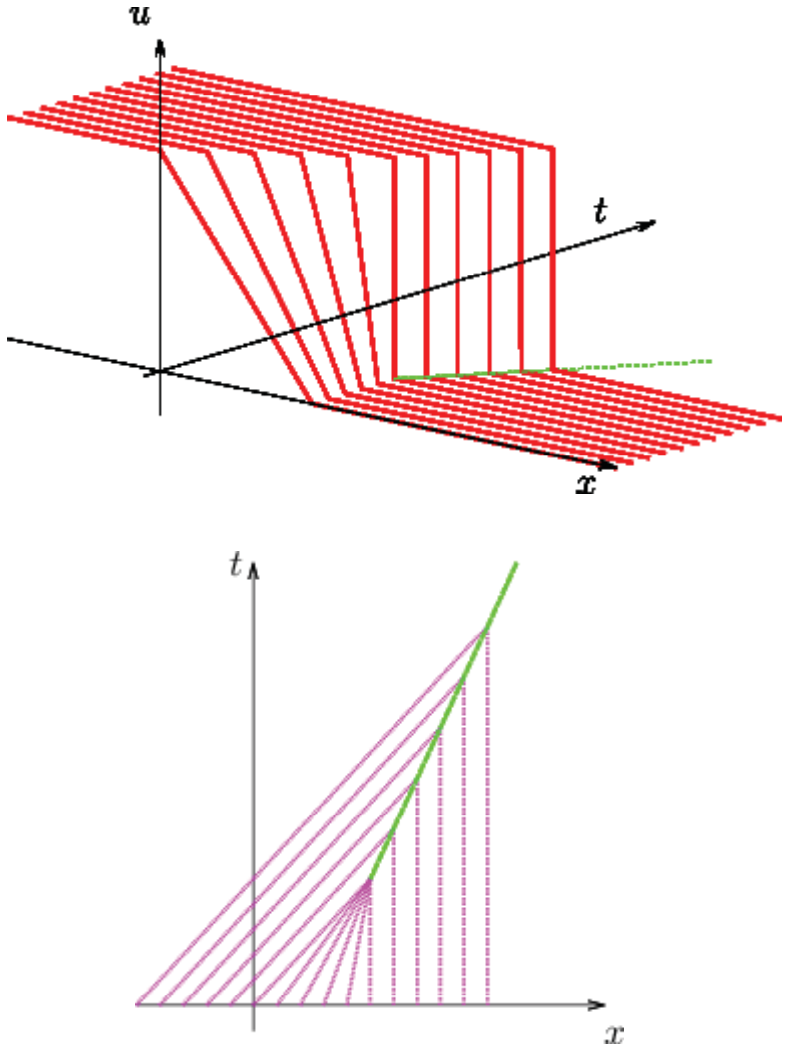
Solvability of Hyperbolic Systems of Conservation Laws

Associate Professor Kuniya Okamoto



Hyperbolic systems of conservation laws described as the first order quasilinear partial differential equations have been extensively studied. The most remarkable feature is that not only this type of equations do not possess the smoothing effects but also the regularities of classical solutions will be lost in finite time even if the initial data are smooth. We introduce the notion of weak solutions which interprets the derivatives of solutions in the generalized sense, then we need to allow the presence of discontinuities in the solutions such as shock waves and discuss the solvability in the wider class. However, in contrast to the single conservation laws, the case of systems has not yet been successfully solved until recently, except for the case that the total variation of initial data is sufficiently close to the equilibrium. In terms of the interaction potential estimates of Glimm type, we study the approximate solvability of a system of conservation laws and the stability of weak solutions even if the total variations of initial data are not small for the presence of large oscillations.

Keywords : Hyperbolic systems, Conservation laws

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